Postulates and Experimental Features in Maxwell's Electromagnetic Equations

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ABSTRACT Ampere's and Faraday's laws are shown to represent, at most, a first approximation of the electromagnetic features they claim to describe. Maxwell's differential equations can therefore be viewed only as axiomatic assertions.

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I. INTRODUCTION

Standard textbooks of classical electromagnetism sometimes [1] axiomatically postulate, from the very outset, Maxwell's electromagnetic equations in their differential form

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi \boldsymbol{J}}{c} + \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} \tag{2}$$

The Lorentz invariance of these equations, together with the wave equation they provide, should allow, in principle, a coherent and causal description of any possible electromagnetic feature.

There exist, however, many phenomena which appear to be much more directly described by the integral (i.e. macroscopic) form of Maxwell's equations:

$$\oint \boldsymbol{E} \bullet d\boldsymbol{l} = -\frac{1}{c}\int_S \frac{\partial \boldsymbol{B}}{\partial t} \bullet d\boldsymbol{S} \tag{3}$$

$$\oint \boldsymbol{H} \bullet d\boldsymbol{l} = \frac{4\pi}{c}\int_S \boldsymbol{J} \bullet d\boldsymbol{S} + \frac{1}{c}\int_S \frac{\partial \boldsymbol{D}}{\partial t} \bullet d\boldsymbol{S} \tag{4}$$

where $d\boldsymbol{S} = \boldsymbol{n}\, dS$, and $\boldsymbol{n}$ is a versor normal to the surface element dS.

It is in such an integral form, indeed, that both Faraday's and Ampere's laws were originally formulated.

The use of eqs.(3) and (4) causes, of course, no conceptual trouble if their complete equivalence with eqs.(1) and (2) is assumed to hold.

It is however our aim to show, in the present work, that such an equivalence is not granted, both for mathematical and physical reasons.

II. THE ROLE OF THE STOKES THEOREM

Let us consider a physical situation where a magnetic field, strictly confined within a limited region (e.g. a long and narrow solenoid) is allowed to vary in time. It is then customary to say that, if we look at an arbitrary loop encircling the region where $\boldsymbol{B} \neq 0$ (and placed even very far from it, in a zone where the magnetic field is and remains negligible) an electromotive force (*emf*) is induced along such a loop, according to the equation

$$\oint \boldsymbol{E} \bullet d\boldsymbol{l} = -\frac{1}{c}\int_S \nabla\times\boldsymbol{E}\bullet d\boldsymbol{S} = -\frac{1}{c}\int_S \frac{\partial \boldsymbol{B}}{\partial t}\bullet d\boldsymbol{S} \equiv -\frac{1}{c}\frac{d\Phi_B}{dt} \qquad (5)$$

where S is any surface covering the loop and Φ_B is the magnetic flux through it. Now, the first two terms of eq.(5) simply represent Stokes' theorem, expressing a geometrical identity when referred to stationary situations. In a time-dependent situation such as the one considered here, however, since eq.(5) connects two distinct and separate regions of space (the loop and the encircled surface *S*) we must expect that the field variation occurring in the solenoid launches an electromagnetic signal, raising in the surrounding space a time -varying electric field

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \qquad (6)$$

with a vector potential $\boldsymbol{A}(\boldsymbol{r},t)$ provided by the expression

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{1}{c}\int_V dV \frac{\boldsymbol{J}(\boldsymbol{r}',\, t-\frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c})}{|\boldsymbol{r}-\boldsymbol{r}'|} \qquad (7)$$

Where $\boldsymbol{J}$ represents the current density distribution in a volume *V* containing the solenoid, and $\boldsymbol{r'}$ is evaluated within the volume element *dV*. Clearly enough, the volume *V* (where the time-varying current density $\boldsymbol{J}$ is distributed,) the surface *S* (through which the flux is computed) and the loop (along which the *emf* is induced) constitute separate and distant regions of space, between which eq.(5) neither establishes nor predicts any kind of propagation, since each one of its terms is computed *at the same time*. Eq.(5) surreptitiously introduces, in other words, an *instantaneous* physical connection. Not surprisingly, the overall result

$$emf = -\frac{1}{c}\frac{d\Phi_B}{dt} \qquad (8)$$

is evidently *non-local*: it asserts, in fact [2], that the electromotive force is *simultaneous* to the magnetic flux variation causing it, wherever it may be generated. In other words, cause and effect (although arbitrarily distant) are declared to be simultaneous, thus implying an *instantaneous*, and therefore *unphysical*, transmission of energy and information.

III. THE ROLE OF EXPERIMENTAL EVIDENCE

Strangely enough, eq.(8), as we already stated, is exactly the form of Faraday's law usually presented as directly provided by the experimental evidence. All standard textbooks, in fact, agree on the fact that it is easily verified by interrupting a conducting loop in a point and inserting an electrometer between its free extremities. As we have seen, however, eq.(8), because of its instantaneous transmission of information, cannot not reflect a physical law. It could represent, at most, a first approximation, where the propagation time is neglected: an excusable mistake for experiments performed in strictly limited spaces, but a quite inexplicable conceptual oversight.

It may be observed that eq.(5) is generally written backwards [2,3], thus claiming to obtain Maxwell's differential equation (1) (which is perfectly **correct**) from the integral one (which, as we have shown, is **incorrect**). Indeed, the non-locality of eq.(8) disappears if we restrict the considered area to a single point.

The same non-equivalence proof, of course, may be shown to hold between eqs.(2) and (4), connecting the electric field variations with the displacement currents.

IV. CONCLUSION

The ambiguous character of Maxwell's equations (experimental inductions or axiomatic postulates?) was already pointed out in previous papers (see, for instance, Refs. [4-7]).

We stress in the present work one more element of puzzling ambiguity. Indeed, if we try, on the one hand, to obtain Maxwell's differential equations (1) and (2) from the integral ones, we start from *incorrect* experimental statements, and make, moreover, an *incorrect* use of Stokes' theorem. The eventual correctness of the result does not absolve us from the erroneous procedure.

If, on the other hand, we simply *postulate* eqs. (1) and (2) (a quite inelegant procedure for an experimental science!) then eqs. (3) and (4) -although, in a way, suggested by the experiment - must be assumed, on second thought, as roughly approximate statements, completely unable to describe, in general, macroscopic physical events.

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