Plasma Physics, Vol. 14, pp. 887 to 891. Pergamon Press 1972. Printed in Northern Ireland

The possibility of producing plasma regions at thermonuclear fusion conditions in a supersonic flow by means of high power electron beam

(*Recieved* 1 *September* 1971)

1. INTRODUCTION

THE suggestion that plasma might be heated to thermonuclear temperatures by insulating it from the walls by means of magnetized, high pressure gas was first made by ALFVÉN and SMÅRS (1960) and FALTHÄMMAR (1961). However, high pressure gas insulation of stationary plasma is less effective than had been anticipated.

Nevertheless, the possibility of using a supersonic flow may revive interest in high pressure gas insulation. The cold gas can be heated using a high power electron beam parallel to the axis of the flow field of the efflux through an annular, supersonic nozzle which supports the pressure differential (Fig. 1).

WINTERBERG (1968) considered the possibility of producing a small thermonuclear explosion of a spherical drop of liquid D–T through its ignition by an intense electron beam. The plasma was considered to be unconfined, and losing energy rapidly by expanding at the sound velocity appropriate to the ignition temperature. However, such an experiment can only simulate a hydrogen bomb explosion on a smaller scale.

The device proposed here is intended to generate a high velocity plasma jet to produce thrust for space vehicle propulsion. As shown in Fig. 1, the hot stationary D–T plasma region is surrounded by supersonically flowing insulating gas. A preliminary calculation of the overall energy balance indicates that this configuration could result in effective nuclear fusion with the energy released in the exhaust gas flow.

2. IGNITION OF A THERMONUCLEAR REACTION BY IRRADIATION OF FLOWING D–T GAS WITH AN INTENSE ELECTRON BEAM

The method of initiating a pulsed fusion reaction of a flowing high pressure D–T gas mixture is based on the recently reported technique (LINK 1967; GRAYBILL and NABLO 1967; CHARBONNIER *et al.*, 1967) in which very intense pulsed electron beams, with power up to 10^{12} W and 1–10 nsec duration, can be generated. The electron beam is extracted by field emission, but for longer times plasma cathodes could be effective if a strong magnetic field is used to prevent arc formation. The energy for the pulsed field-emission discharge is normally obtained from a large capacitor bank connected as a high voltage Marx generator.

The first condition, necessary for thermonuclear ignition, is that the temperature in the high pressure flowing D–T gas must be raised to about 20 keV ($T \approx 2{\cdot}32 \times 10^8$ °K). To obtain a reasonably high fusion power density the stagnation pressure of the plasma is maintained at 10^9 dyn cm^{-2}.

The D–T gas would be injected from an annular nozzle with an equivalent throat cross-section about 10 mm^2. The supersonic flow becomes subsonic, with a normal shock, by interacting with the electron beam; the flow is then accelerated during the heating process. To heat the resulting mass flow of D–T gas ($\dot{m} \simeq 220$ g sec^{-1}) to 20 keV requires a power about $3{\cdot}4 \times 10^{11}$ W.

The rate of thermonuclear power generation per unit volume is given by

$$P_{TD} = n_D n_T \overline{\sigma v} \times 5{\cdot}6 \times 10^{-13}\ \mathrm{W\,cm^{-3}},$$

where the product $\overline{\sigma v} = 4{\cdot}3 \times 10^{-16}$ cm^3 sec^{-1} for 20 keV D–T plasma. If

$$n_D = n_T = 10^{17}\ \mathrm{cm^{-3}}, \qquad P_{DT} = 2{\cdot}4 \times 10^6\ \mathrm{W\,cm^{-3}}.$$

The calculation of the reacting volume is very difficult since it involves determining the entire flow field. If we take the plasma pressure to be constant over the reaction length, we obtain a reaction volume about 3×10^6 cm^3. Let us take a lower value, i.e. a reaction volume $V = 10^6$ cm^3, so that the total power produced is:

$$P_{DT}V = 2{\cdot}4 \times 10^{12}\ \mathrm{W}.$$

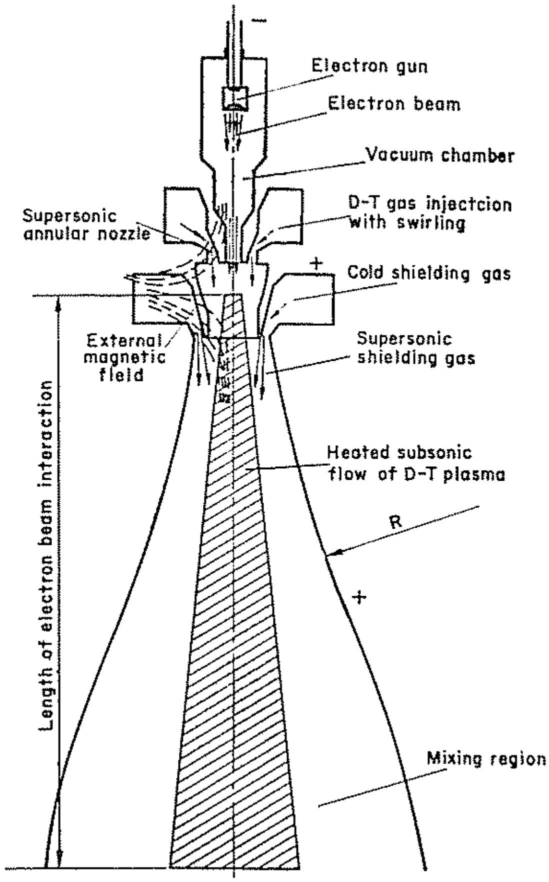

FIG. 1.—Proposed device for electron-beam generation of T–D plasma with shielding by supersonic flow field.

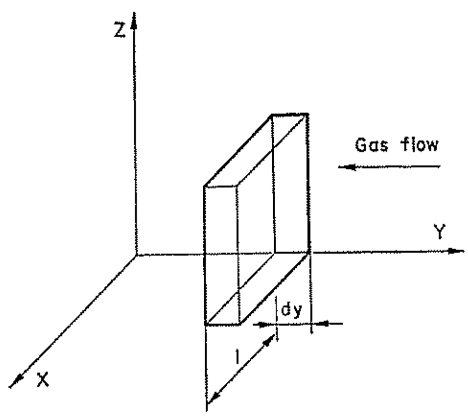

FIG. 2.—Elemental volume and reference axis for equation (3) numerical calculations.

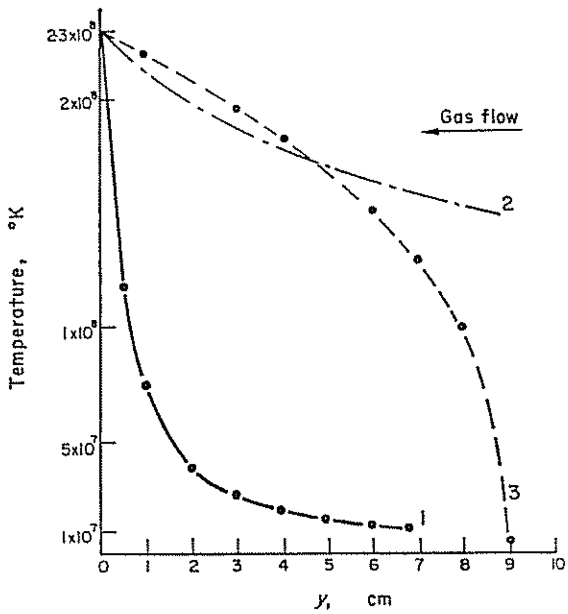

FIG. 3.—Diagrams of temperature distribution $T(y)$ of the incoming T–D plasma. The curves are obtained from numerical calculations using equation (3) for three different expressions of the thermal conductivity $K(T)$ (W/deg.cm). 1, $K = 2{\cdot}8 \,.\, 10^{-24} n_i^2 T^{-1/2}$ derived from SPITZER (1962), equation (5-53) for a very low magnetic field (B = 3); 2, $K = 2{\cdot}4 \,.\, 10^{16}\, T^{-5/2}$ derived from ALFVÉN and SMÅRS (1960) for $H = 3$; 3, $K = 7{\cdot}518 \,.\, 10^{-16}\, T^{5/2}$ derived from SPITZER (1962), equation (5-47) in the absence of a magnetic field (lowered of a factor 10^{-4} to have values of the same order of magnitude obtained with the previous expressions).

An external magnetic field of $(1{\cdot}5$–$2) \times 10^5$ G must be provided by a superconducting coil system. The mean power conducted away per unit length can be estimated, using an approximation (FALTHAMMAR, 1961) for the conduction losses, to be 100 kW cm^{-1}. For a reaction length of a few meters the total heat conduction loss (transverse to the magnetic field) is $\sim 10^7$–10^8 W. Thus the conduction losses, being several orders of magnitude smaller than the thermonuclear power produced, do not affect the energy balance of the heating processes.

The bremsstrahlung loss is given by

$$P_{br} = 5{\cdot}3 \times 10^{-31} n^2 T^{1/2}\ \text{W cm}^{-3} \qquad (T \text{ in keV})$$
$$= 2{\cdot}36 \times 10^4\ \text{W cm}^{-3}$$

for a 20 keV D–T plasma. Thus the bremsstrahlung and other radiation losses make a slight contribution to the total losses.

The beam energy must be supplied to the flowing D–T gas in a time much shorter than its time spent in the duct. With a maximum duct length of 10^3 cm, this cannot exceed 10^{-3} sec. Electrons with several MeV energy have a long range even in high pressure gas. However, the situation is better than that for single particle interaction since an intense beam can lose energy by processes arising from the counter streaming instability (BUNEMAN, 1959). WINTERBERG (1968) derived an expression for the distance λ over which the beam loses its energy:

$$\lambda \simeq 1{\cdot}4(n/n_b)^{1/3}(c/\omega_{pe}) \qquad (1)$$

where n is the plasma density, n_b the beam density, ω_{pe} is the electron plasma frequency and c the velocity of light.

LINK (1968), working with electron beams up to 10^{12} W and atmospheric pressure air, found some experimental evidence of strong energy transfer. Nevertheless equation (1) could not explain this interaction since it leads to unreasonably low values of λ.

The dissipation associated with equation (1) is known to lead primarily to electron heating in the D–T gas. The ions are then heated by electron–ion collisions with equipartition time given by:

$$t_{ei} = 3 \times 10^{8} T_e^{3/2}/n_e \text{ sec}, \tag{2}$$

where T_e is the energy of the injected electrons (in keV) and n_e the initial electron density of the D–T plasma. Taking $n_e \simeq 10^{20}$ cm^{-3} gives $t_{ei} = 5 \times 10^{7}$ sec. This is short enough to heat the ions to fusion temperature within the plasma transit time.

Because of the power transferred from the inner core, the flow field streamlines of the outer shielding gas will be deflected, with a radius of curvature given by Crocco's equation for non-isentropic flow with the addition of heat:

$$R = 2q^2/\left(T\frac{\partial s}{\partial n} - \frac{\partial h^\circ}{\partial n}\right)$$

where the partial derivatives $\partial/\partial n$ are taken in the direction normal to the streamline, q is the gas velocity, s is the gas entropy, and h° the total enthalpy: $h^\circ = h + q^2/2$. The pressure in the insulating supersonic flow can be maintained at the necessary level provided the duct wall has suitable curvature.

3. STEADY FUSION CONDITION

The conditions for a self-sustaining fusion reaction, ignited by an injection pulse, depend on heating the incoming cold D–T gas by some fraction of the thermonuclear power generated. The possibility of continuous, self-sustained operation requires therefore that there is a sufficiently large axial thermal conductivity to heat the incoming gas to 20 keV. As already stated, an axial magnetic field is produced throughout the fusion volume by an external superconducting solenoid. The strength of this field should be at least 2×10^{5} G (a value which should be achieved by future technology). With the axial magnetic field reversed in the injection section a region of almost zero magnetic field can be obtained.

For the sake of simplicity let us consider a model in which the gas flows in the $-y$ direction and all physical properties are constant in the x and z directions (see Fig. 2). In this way we may suppose that the counterflow heat transmission is only in the y-direction, the temperature gradient in this direction being large near some plane at which the fusion reaction is assumed to start. The increase per unit time dQ of the quantity of heat contained within the elemental volume of Fig. 2 is given by the difference between the heat entering and leaving the volume:

$$\mathrm{d}Q = \frac{\mathrm{d}}{\mathrm{d}y}\left[K(T)\frac{\partial T}{\partial y}\right]\mathrm{d}y$$

where $K(T)$ is the thermal conductivity (in W deg^{-1} cm^{-1}) of the plasma at temperature T. In calculating the temperature distribution of the incoming D–T plasma we assume that fusion power is generated at temperatures above the ignition temperature T_i. This is a pessimistic assumption, save in practice when the plasma undergoes fusion, at a slower rate, below the ignition temperature, which is taken at $2{\cdot}3 \times 10^{8}$ °K.

In a steady state the gas flowing across the volume element (with a mass flow rate per unit area equal to $\rho(T)v(y)$, where $\rho(T)$ is the gas density, $v(y)$ (its velocity) increases its enthalpy $h(T)$ by an amount

$$-(\partial h/\partial y)\,\mathrm{d}y = -\left(\frac{\partial h}{\partial T}\frac{\partial T}{\partial y}\right)\mathrm{d}y.$$

If we disregard the change in flow area we can take the product ρv = constant.

The energy equation for the heating process in the flow becomes:

$$-\rho v\left(\frac{\partial h}{\partial T}\frac{\partial T}{\partial y}\right)\mathrm{d}y = \frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{\partial T}{\partial y}K(T)\right]\mathrm{d}y,$$

$$\frac{\mathrm{d}^2T}{\mathrm{d}y^2}K(T) + \left(\frac{\mathrm{d}T}{\mathrm{d}y}\right)^2\frac{\mathrm{d}K}{\mathrm{d}T} = -\rho v\frac{\partial h}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}y} \tag{3}$$

$$\frac{\mathrm{d}^2T}{\mathrm{d}y^2} = -\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right)^2\frac{\mathrm{d}K}{\mathrm{d}T}\frac{1}{K} - \rho v\frac{\partial h}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}y}\frac{1}{K}$$

This second order non-linear differential equation can be integrated numerically if we know the initial conditions at $y = 0$. We can find the first derivative of the temperature at the interface between the fusion region and the non-reacting incoming gas, since the latter is heated only by thermal conduction from the fusion region. Hence the heat flux $K(T_i)(dT/dy)_{y=0}$ must balance the heat carried in the opposite direction by the heated flowing plasma, which reaches the interface at the desired ignition temperature, i.e.

$$K(T_i)(dT/dy)_{y=0} = \rho v(h_i - h_0).$$

Therefore

$$(dT/dy)_{y=0} = \rho_0 v_0(h_i - h_0)/K(T_i)$$

and

$$T = T_i$$

are the two initial conditions required for integrating equation (3).

The form of the thermal conductivity used depends very much on the model adopted. For example, if we consider the case of plasma with no magnetic field, $K \propto T^{5/2}$, but in the presence of a magnetic field $K \propto T^{-5/2}$ (ALFVÉN and SMÅRS, 1960) or $\propto T^{-1/2}$ (SPITZER, 1962). These uncertainties are reflected in the numerical integration of equation (3).

Figure 3 shows curves of $T(y)$ calculated for the various forms for $K(T)$ for values of T near T_i. The experimental study of thermal conductivity at high temperatures and pressure, both with and without magnetic field, is of fundamental importance to the future feasibility of continuous thermonuclear fusion reactions shielded by a supersonic gas flow.

Finally, it should be noted that the stability of plasma surrounded by a dense gas differs from one surrounded by vacuum or rarefied gas. The flute instability cannot occur when the surrounding gas is supersonic, nor if its velocity is less than that of the central subsonic flow.

4. CONCLUSIONS

In the mode of operation considered here the plasma temperature cannot, for practical reasons, be maintained by continuous injection if a steady-state fusion reaction is required. To reach conditions necessary for a self-sustained fusion reaction, ignited by pulsed injection, the recoverable nuclear power must exceed the total losses. The so-called direct losses are those associated either with the escape of bremsstrahlung and other radiation, or with the escape of charged particles from the reaction region. For a fusion reactor used as a rocket motor both these direct losses actually contribute to the generation of thrust, firstly by heating the surrounding insulating gas which is accelerated in the divergent nozzle, secondly by the escaping energetic particles themselves.

With the proposed device it appears possible to ignite a short duration fusion reaction although the Lawson criterion ($n\tau \geq 10^{14}$ sec cm^{-3}) is not completely satisfied (in our case $n\tau \leq 10^{14}$). Some refinements of the flow field model are necessary to increase the value of $n\tau$ to establish steadystate fusion reaction after the electron pulse.

Istituto di Fisica,
Facúlta di Ingegneria dell'Universita di Ancona 60100,
Italy

R. GIOVANELLI

REFERENCES

ALFVÉN H. and SMÅRS E. (1960) *Nature, Lond.* **801,** 188.
BUNEMAN O. (1959) *Phys. Rev.* **115,** 503.
CHARBONNIER F. M., BARBOUR J. P., BREWSTER J. L., DYKE W. P. and GRUNDHAUSER F. J. (1967) *IEEE Trans. Nucl. Sci.* **NS-14,** 789.
FÄLTHAMMAR C. G. (1961) *Phys. Fluids* **4,** 1145.
GRAYBILL S. E. and NABLO S. V. (1967) *IEEE Trans. Nucl. Sci.* **NS-14,** 782.
LINK W. T. (1967) *IEEE Trans. Nucl. Sci.* **NS-14,** 777.
WINTERBERG F. (1968) *Phys. Rev.* **174,** 212.
SPITZER L. (1962) *Physics of Fully Ionized Gases.* Interscience, New York.