# STRETCHED HOLLOW RAIL SUSPENSION BRIDGE

Raffaele Giovanelli

Università di Parma - Parma (ITALY)

**ABSTRACT - We propose here the design of a suitable bridge-chain, providing a new mass transport system which strongly reduces the need of rigid and heavy structures. This system is based on a highly deformable and light structure, provided with suitable damping devices, and able to sustain special and fast vehicles, whose mass is comparable with that of the bridge itself.**

*Keywords:* Suspended bridge, monorail, tensioned structures, mass transportation

## 1- Introduction

Traffic problems in major cities around the world during the last two decades have presented important needs of new transportation systems [1].

Many kinds of monorail systems, for instance, turned out to provide low-cost and flexible solutions for already existing urban centers.

An important drawback of these structures is often given by their weight, requiring bridges with very short span, and therefore with a large number of pylons.

A most improved solution can be given by cableway systems composed of suspension catenary cables that hold track cables, as first proposed by Gerhard Müller [2] in 1973. Such a system was tested in Germany, and can span distances of several hundred meters between support masts, to be compared with the distances of 10 to 15 meters allowed by rigid rail structures (monorail).

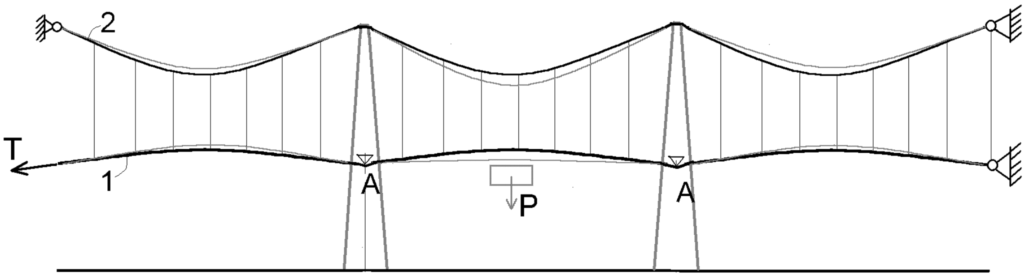



Fig. 1. Scheme of Müller's system [2]. The critical points (A) of such a system are the ones where the rail(1) is held at a fixed height, thus opposing to the sustaining rope (2).

However, while cable-supported rails can span to greater distances than rigid rails, the vehicle speed in cable-suspended rails turns out, unfortunately, to be quite low.

This is due both to the downward bow of the rail between the supports and to the sudden upward bending of the rail in the region of the supports (Fig. 1, points A), while the travel path in Müller's design was expected to be approximately straight.
In spite of the fact that the sag during travel appears to be reduced or eliminated by this technique, experiments with it have shown that it does not have the desired effect, namely, it does not allow a substantial increase in speed [3].
The disadvantage is that, in the unloaded state of the track at the pylons, crimps are formed in the track in the regions where it is suddenly deflected upward by the negative sag arrangement. This is because at the pylons a downward reaction force must be applied to the track in order to keep it from moving upward.
Even when the vehicle is travelling along the track, and its load balances the upward force producing the upward sag, the crimping of the track at the support (pylon) tends to remain. The crimped parts of the track reduce the maximum speed which can be developed therealong, giving rise to excessive wear. Notwithstanding these shortcomings, the system proposed by Gerhard Müller in 1973 [2] provided the first non standard approach of a class of new aerial transportation systems [3, 4, 5]. A design performed according to Müller's idea is shown in Figs. 2, 3, 4.

Fig. 2. Artistic view of a transportation system based on Müller's project [2], with a pylon containing a vehicle stop.

Since, in Müller's approach, the weight of the vehicle creates sags along the track cable, he proposed to address these problems by pre-tensioning the track cable systems. Thanks to his suspension system, track wire ropes were tensioned and bowed upward when not weighted by vehicle, thus creating a more rigid and stable structure.
We present here, in alternative to rigid mono-rail systems, a light and highly flexible structure in the form of a bridge-chain, able to support heavy and high velocity vehicles, duly adapted to a hollow suspended rail. The critical part of the rail at the points A (Fig.

1) is made with a semi-rigid structure (not shown here) that permits the necessary sliding of the rail. A further advantage of this approach is allowed by the fact that the most important oscillations are the ones due to the mass of the vehicles. Because of the high speed of the vehicles themselves, and of the lightness of the structure, these oscillations have a period which turns out to be comparable to (or even longer than) the transit time of the vehicles. The details of this new hollow rail shall be shown in Appendix C, Fig. 12.

The oscillations due to the deformation induced by the weight of the vehicles and triggering  the behaviour of the system are quenched by suitable (active and semi-active) dampers, placed between the suspension cable and the hollow rail (Fig 3).

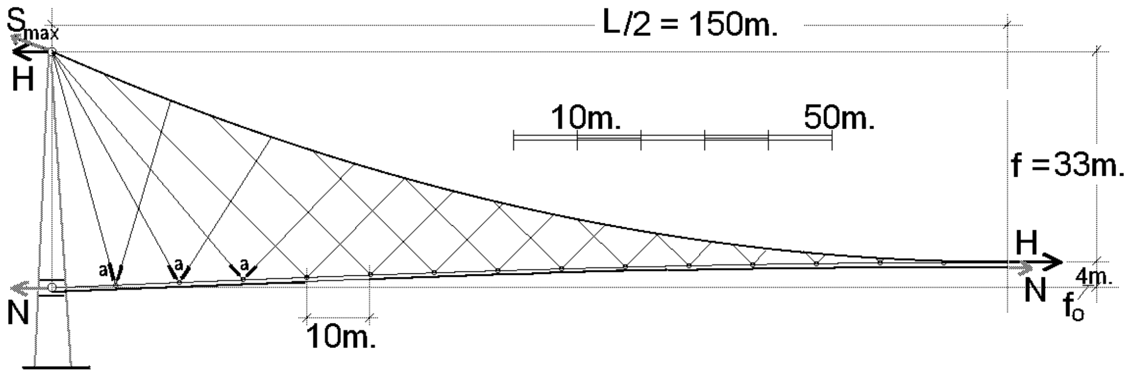



Fig. 3 General view of the left half of the bridge, submitted to the stresses H, N, $S_{max}$. The positions a) indicate the active dampers [6]. The hangers are inclined at 45°, twined in such a way that the only component of the suspension force at the points of sliding anchorage is normal to the girder (the theory is carried out considering the hangers be vertical).

## 2 – Theory

We shall complete here the classical and well-established theory of a suspension bridge [7, 8] by taking also into account the traction strain (whose horizontal component we shall call *N*) imposed by the boundary conditions to the suspended girder (hollow rail), thus creating a tenso-structure whose rigidity exceeds the one due to the dead load.

The classical theory of a suspension bridge [8,9] simulates its structure as a single element including the sustaining ropes and an almost straight girder, where the effect of the deformation of the sustaining ropes is largely prevailing. The equation of the suspended bridge has therefore the same form as the general differential equation of a simple beam which is simultaneously loaded with an axial (fictitious) traction force at the ends and a transverse vertical load.

The present model, however, being based on a curved girder, must also take into account its traction, due to the real axially imposed tension T, which are liable to increase the load on the sustaining ropes (Fig.5).   We shall call, in the following:

$q$ the structure weight per unit length,

$p(x)$ the live weight distribution along the girder,

$H$ the initial horizontal component of the sustaining ropes tension,

$h$ the additional horizontal component of $H$ when $p(x)$ is applied to the girder,

$T$ the traction force applied to the girder,

$N$ the horizontal component of the traction force $T$ applied to the girder,

$q_N$ the force per unit length due to T and to the girder configuration,

$y(x)$ the initial profile of the sustaining ropes,

$w(x)$ the additional deformation of the ropes, which is assumed to be equal for the girder, by neglecting the lengthening of the hanging cables,

$L$ is the bridge span,

$w_o(x)$ the girder profile in the absence of external load,

$J\ [m^4]$ is the total inertial momentum of the girder cross section (see Fig. 3), and finally

E = 206 [Gpa] is the Young modulus of the steel of the ropes,

$P = \int_o^L p(x)\, dx$ is the total weight of the vehicle.

The horizontal component $H$ is due both to the structural constant weight $q$ and to the effect of the traction $T$ applied to the girder. We assume $y(x)$ to be positive below the *x-* axis (see Fig. 5). The differential equation of the space profile of the sustaining ropes *in the absence of external loads* may be written in the form [8]:

$$H \frac{d^2 y}{dx^2} = -q - q_N \qquad (1)$$

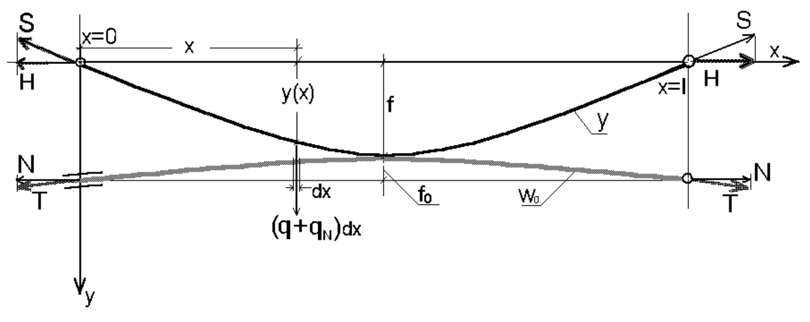



Fig. 4. Configuration of the structure imposed by its weight (q(x)= q ≅ const) and by the load $q_N$ , in the absence of an externally imposed load

Assuming an initial profile of the girder of the form of a reversed catenary, the girder shall impart to the sustaining ropes (besides its dead load, which is already included in *q*) a uniform extra load given by

$$q_N = -N \; \frac{d^2 w_o(x)}{dx^2} \quad . \qquad (2)$$

Therefore, an upward extra load per unit length $-q_N$ shall be required by the geometry of the girder.

The space profile of the rail girder *after the application of the live load* (Fig. 6) shall be given by the function

$$w_t(x) = w(x) - w_o(x) \; ,$$

where $w_o(x)$ is the initial profile, and w(x) is the deformation of the girder (in the positive y direction, to be determined in the following) due to the external load p(x), and to be added to the profile y(x) of the sustaining ropes (to which it is transmitted by the hangers).

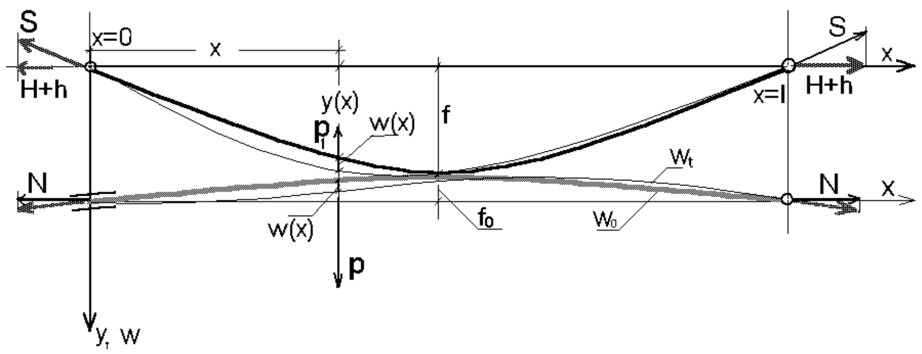



Fig. 5. Configuration of the structure after the application of the load (at x = a).
The configuration in the absence of load is represented by the heavy line; the light line represents the shape of the structure deformed by the load P.

The equation of the sustaining ropes, in its turn, immediately follows from eq.(1), in the form [8]:

$$(H+h)\frac{d^2(y+w)}{dx^2} = -q - q_N - p_1 \qquad (3)$$

where (H + h) is the new total horizontal component of the ropes tension and $p_1$ is the fraction of the total live load p supported by the ropes. The girder shall support the load ($p - p_1$), and its profile shall be the solution of the equation

$$E\ J\ \frac{d^4w}{dx^4} - N\ \frac{d^2w_t}{dx^2}\ = p - p_1 - q_N \qquad (4)$$

where $-q_N$ (gven by eq.(2)) is the constant force per unit length impressed by the sustaining ropes on the girder. As far as the load $-p_1$ is concerned, its value is obtained from eqs.(3) and (4) in the form

$$-p_1 = q + q_N + (H+h)\frac{d^2(y+w)}{dx^2} = q + q_N + H\frac{d^2y}{dx^2} + +h\frac{d^2y}{dx^2} + (H+h)\frac{d^2w}{dx^2}$$

by means of eq.(1) we get now

$$h\frac{d^2y}{dx^2} = -h\,\frac{(q+q_N)}{H}$$

so that

$$-p_1 = (H+h)\frac{d^2w}{dx^2} - \frac{h}{H}(q+q_N)$$

Replacing now this value into eq.(4) we get

$$E\ J\ \frac{d^4w}{dx^4} - (H+h+N)\frac{d^2w}{dx^2} + N\ \frac{d^2w_o}{dx^2} = \ p - \frac{h}{H}(q+q_N) - q_N$$

Making use of eq.(2), we finally get the basic suspension bridge equation with tensioned girder in the form

$$E\ J\ \frac{d^4w}{dx^4} - (H+h+N)\frac{d^2w}{dx^2} = \ p - \frac{h}{H}(q+q_N) \qquad (5)$$

The axial tensile force (H + h) acting on the sustaining ropes is a fictitious term, and produces therefore no tensile stresses in the stiffening girder.
The solution of eq.(5) may be shown to be given by the function

$$w(x) = w_1(x) - \frac{h}{H}\ (q+q_N)\,w_2(x)\,, \qquad (6)$$

and are given in Appendix A.
Let now $L_r$ be the length of the rope submitted only to the dead load of the structure [8]

$$\Delta L = \frac{1}{H}\int_o^L \left[1+(y')^2\right]^{1/2}\ dx$$

and let $\Delta L$ be the increase of $L_r$ due to the external load p(x).
The solution of eq.(5) requires the determination of the parameter *h*, representing the increase, due to the external load p(x), of the stress acting on the sustaining ropes.
Because of *h*, the profile of the sustaining rope passes from *y(x)* to *y(x) + w(x).* Recalling now that, for ε < 1,

$$[1+(x+\varepsilon)^2]^{1/2} - [1+x]^{1/2} \approx x\,\varepsilon\,[1+x^2]^{1/2} \quad ,$$

we get

$$\Delta L = \int_o^L \left[1+(y'+w')^2\right]^{1/2} \cdot dx - \int_o^L \left[1+y'^2\right]^{1/2} \cdot dx = \int_o^L y'\,w'\,[1+(y')^2]^{-1/2}\,dx$$

By a simple integration by parts we get

$$\Delta L = [w\,y'\,(1+(y')^2)^{-1/2}]_0^L - \int_o^L w\,y''[1+(y')^2]^{-3/2}\,dx \quad .$$

By observing now that w(0) = w(L) = 0, approximating $[1+(y')^2]^{-3/2} \approx 1$ and recalling eq.(1), we get

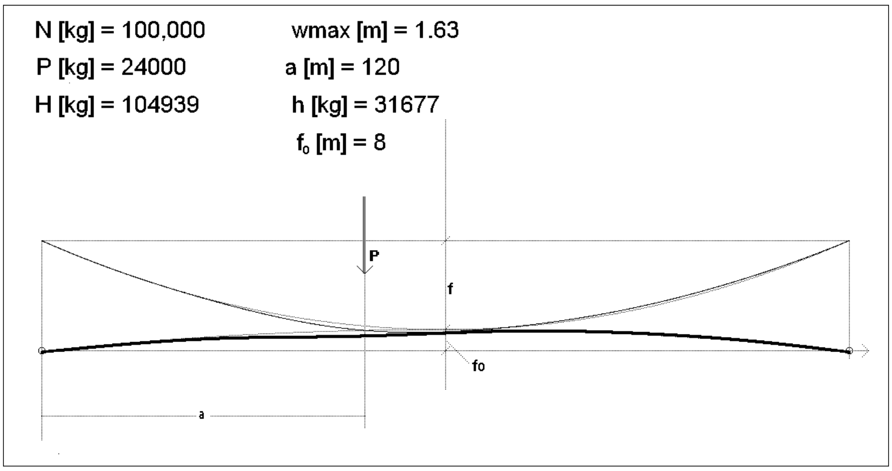
$$\Delta L = \frac{1}{H}\int_o^L [w(x)\,(q+q_N)]\,dx$$

Recalling now that $\Delta L \cong h \cdot L / E \cdot A$, we finally obtain the relation

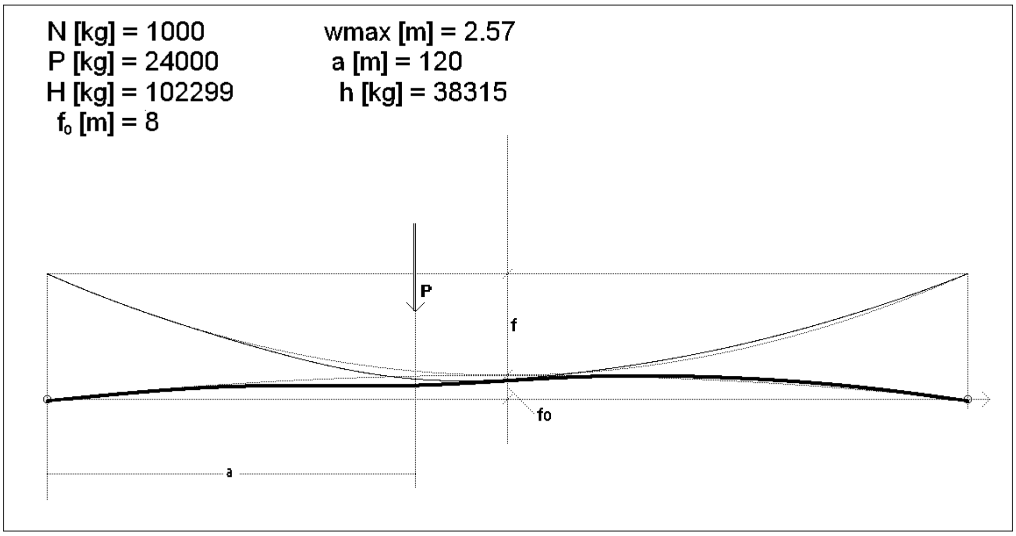
$$\int_o^L w(x,h)\,dx \approx \frac{H\;\;h\;\;L_r}{A\,E\,(q+q_N)} \tag{7}$$

from which the value of *h* ( to be introduced into eqs.(A3) , (A4) and (6)) may be straightforwardly deduced by numerical means. The static bridge deformations shown in Figs. 7 a,b,c are obtained with the load conditions thereby specified. Eq. (6) provides now the complete expression of the deformation both of the rope and of the girder, allowing therefore for static loads the general description of stress and strain to which the entire bridge structure is submitted. In the following, the total dead load per unit length is given by *q = 300 [kg/m];*

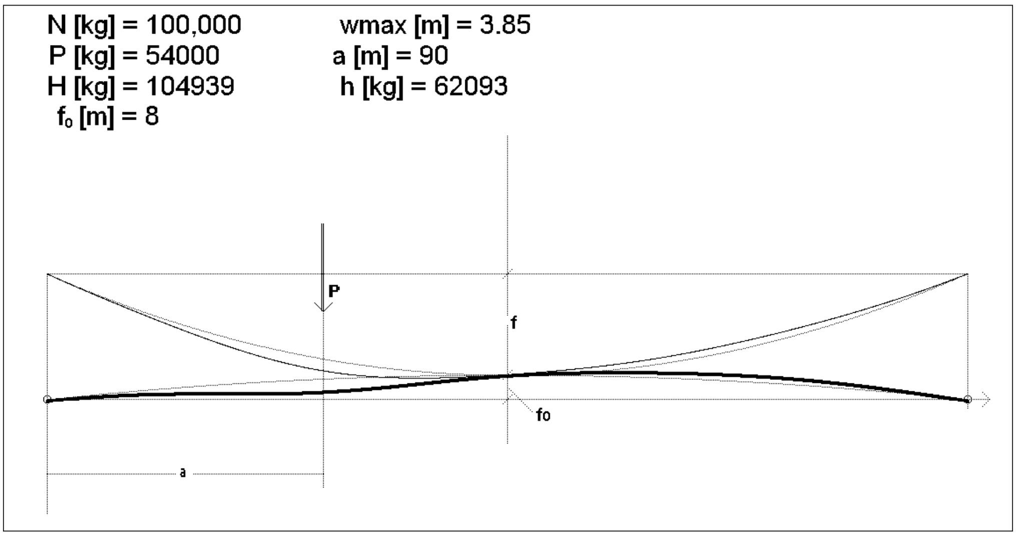
the cross-section of the two sustaining ropes is *4061* $[mm^2]$*;* the diameter of each rope is *51 mm*; the cross-section of the two ropes included in the girder (Fig. 3)is *14137* $[mm^2]$; the values tentatively employed for the moment of inertia of the girder cross-section are $9 \cdot 10^{-4}$ and $2 \cdot 10^{-3}\,[m^4]$, respectively; the Young modulus E for the steel ropes is 206 [Gpa]. Consequently, $E \cdot J\,[kg \cdot cm^2] = 1.85 \cdot 10^8\,[Nw \cdot m^2]$ or $4.116 \cdot 10^8 [Nw \cdot m^2]$, considering for the cross section of the entire girder an area-equivalent method that take account of the non-steel part of the structure.




6a

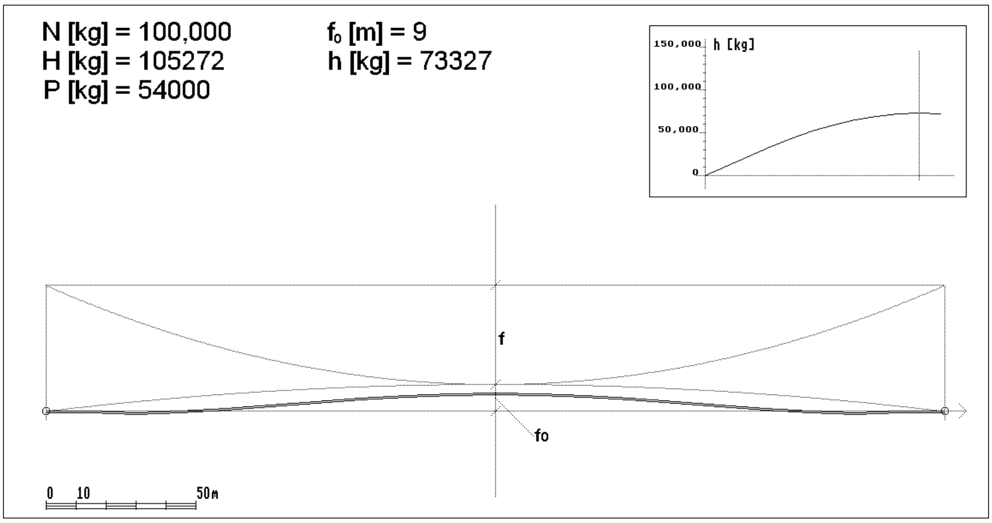



6b

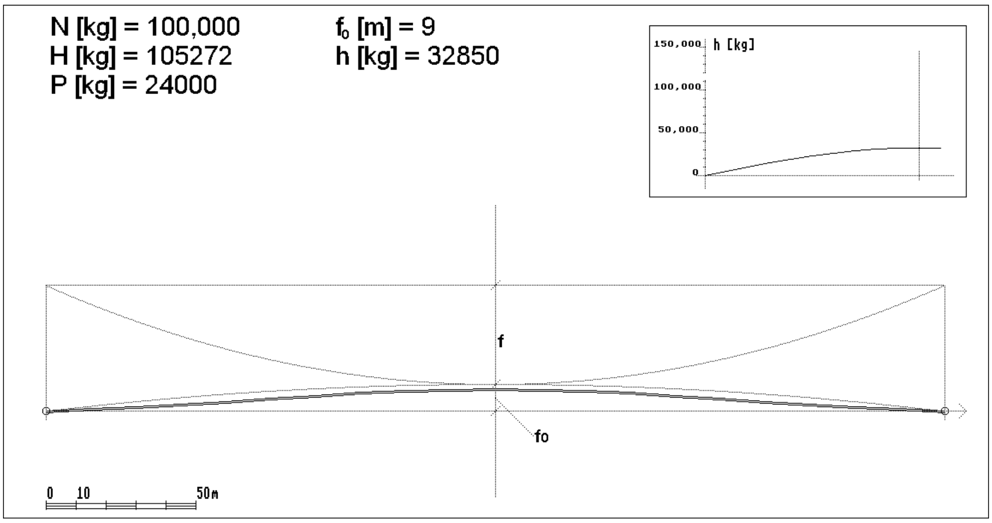



6c

Fig. 6a, b, c. The configuration of the whole structure is obtained (in static conditions) from eqs. 6), A3), A4) and 7) for some positions of the external load P and for different values of the horizontal component, N, of the tension T applied at the ends of the girder rail. $E \cdot J = 1.85 \cdot 10^8$ [$Nw \cdot m^2$]




7a




7b

Fig. 7 a, b. The quasi-static trajectory of the load is obtained by repeating the calculation of Fig.1 for a set of load positions all along the girder ($E \cdot J = 1.85 \cdot 10^8$ [Nw m$^2$]).

**In conclusion:** The girder is a beam under the simultaneous action of a transverse and an axial (both fictitious and real) loading. Obviously the maximum stress in the external fibres of the girder cross section (Fig. 3) can be calculated knowing the moment of inertia *J* of the section about the central axis normal to the plane of bending. The local bending moment *M(x)* is given by $M = - w'' EJ$ , where $w''$ is obtained from *w(x).*

**3 - Time-dependent theory**

The structure (Fig. 3) is submitted to several vibration modes. In our design every hanger is endowed with a semiactive damper, thus disconnecting from one another the vibrations of the two main elements of the bridge (the girder and the ropes). The high frequency vibrations of the girder, in particular, turn out to be almost completely quenched also because of some actuators of an active damping system (see Fig.3), positioned at the ends of the stay cables [6]. The girder rail shall therefore behave as a beam resting (with damping) on elastic foundations and subjected to an axial tensile force $N_t$.

The analysis of the girder vibrations shall be faced after the first version of the detailed project. We shall consider here only the vibration frequencies of the structure as a whole [10]. We shall neglect, in particular, the fact that an end of the girder slides horizontally, allowing (by the traction of the hydraulic cylinders) to hold constant the tensile force acting on the girder.

All the ends of the sustaining ropes shall therefore be considered to be fixed by suitable hinges.

Ref. [11] takes into consideration an aeroelastic model of a suspended footbridge, which has two suspension cables and a bridge deck.

The theoretical values of the natural frequencies, and the natural modes of vibrations of the coupled system (consisting of the suspension ropes and the bridge deck with prestressing tendons), were determined from the theory of pre-stressed networks. This theory makes it possible to determine the quantities for planar vibrations, provided that the suspension points of the ropes remain motionless during the vibrations. The vibration mode requiring the lowest amount of energy turns out to have the frequency

$$f_j = \frac{1}{2}\ [\frac{g}{q}(\frac{N_t\ j^2}{L^2} + \frac{E\ J\ j^2\pi^2}{L^4})\ ]^{\frac{1}{2}} \qquad ,$$

coinciding with the expression given in Ref.[7] for a beam submitted to the same tensile force.

In order to describe the time dependent behaviour of the bridge, we should introduce now suitable inertial terms into eq.(5). We may avoid, however, this procedure by recalling that the general differential equation of a simple beam, loaded both with an axial force $\overline{N}$ and with a transverse load $\bar{p}(x,t)$, may be written in the form:

$$E\ J\ \frac{d^4w}{dx^4} - \overline{N}\ \frac{d^2w}{dx^2} + \frac{q}{g}\ \frac{d^2w}{d\,t^2} = \bar{p}(x,t)\,, \qquad (8)$$

and by observing that this equation, when neglecting its time dependence, reduces to our eq.(10) by means of the simple substitutions

$$\overline{N} = H + h + N \qquad \text{and} \qquad \bar{p}(x,t) = p(x,t) - \frac{h}{H}(q + q_N)\ .$$

We may therefore analyse the oscillation frequencies of the beam described by eq.(8), and extrapolate them to our suspension bridge.

We can assume the equation (5) as the description of a structure formed actually by a single beam [7] where also the dynamic behaviour is that of a single beam with all the fictitious loads applied. This can be accepted if we assume the hangers between the stiffening girder and the suspension cable to be rigid. The frequencies of transverse oscillations of a single beam described by equation (8) are:

$$\omega_n{}^2 = \frac{n^2\pi^2}{L^2} \cdot \frac{N \cdot g}{q} + \frac{n^4\pi^4 E\,J\,g}{q\;L^4}. \qquad (9)$$

The general dynamical equation of the whole structure may be written in the form

$$E\;J\,\frac{d^4w}{dx^4} - (H + N + h(t))\,\frac{d^2w}{dx^2} + m\,\frac{d^2w}{d\,t^2} = \;p(x,t) - \frac{q + q_N}{H}\,h(t)\,, \qquad (10)$$

where h(t) is the time dependent variation of the tension H of the ropes, p(x,t) represents the load of the vehicle displacing along the beam, and $m = q/g$ is the mass of the bridge per unit length. The function p(x,t), in its turn, may be expressed [8] by means of a Fourier expansion in the form

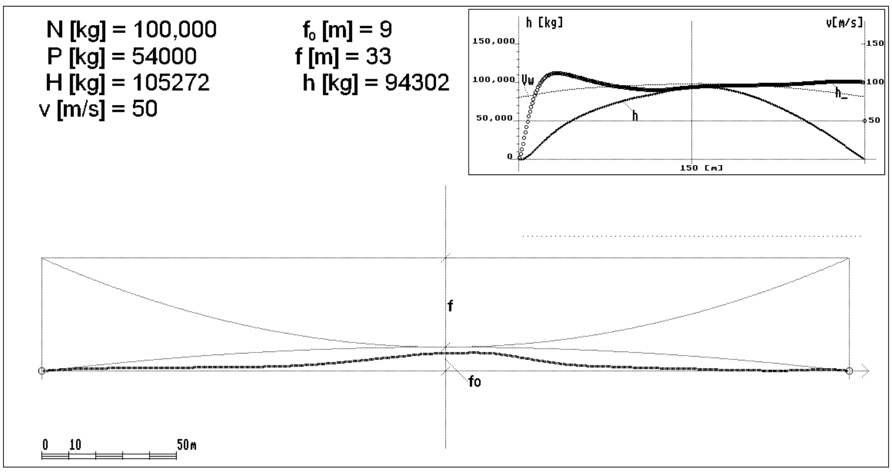
$$p(x,t) = \sum_{n=1}^{\infty} p_n\;sin(\frac{n\,\pi\,x}{L})\;sin(\frac{n\,\pi\,v\,t}{L}) \qquad (11)$$

where $v$ is the velocity of the vehicle, v·t is the distance from the origin of the load P at the time t, and the coefficient $p_n$ turns out to be a constant: $p_n = 2\,P/L$.
As shown by eq.(11), we may associate [7] to the motion of the vehicle the frequency $\omega = \pi v / L$ at which the load *P* acts on the structure. Making use of eq.(11), eq. (10) becomes:

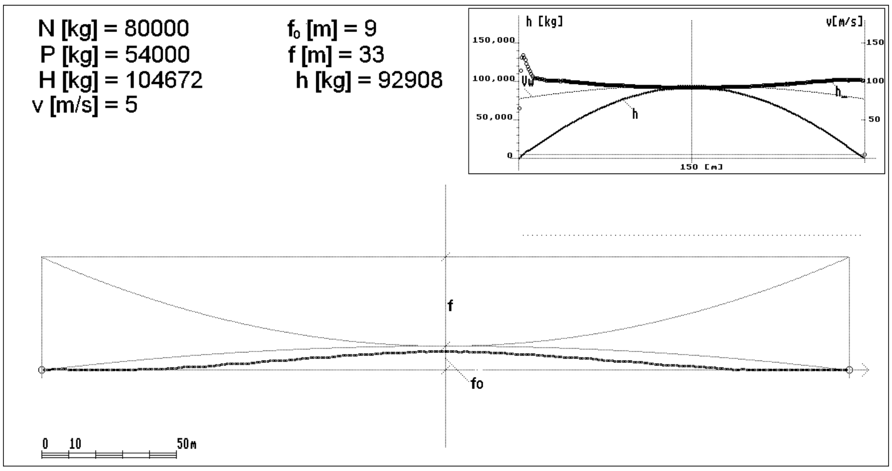
$$EJ\,\frac{d^4w}{dx^4} - (H + N + h(t))\,\frac{d^2w}{dx^2} + m\,\frac{d^2w}{d\,t^2} = \sum_{n=1}^{\infty} p_n\;sin(\frac{n\,\pi\,x}{L})\;sin(n\omega t) - \frac{q + q_N}{H}\,h(t)$$

(12)

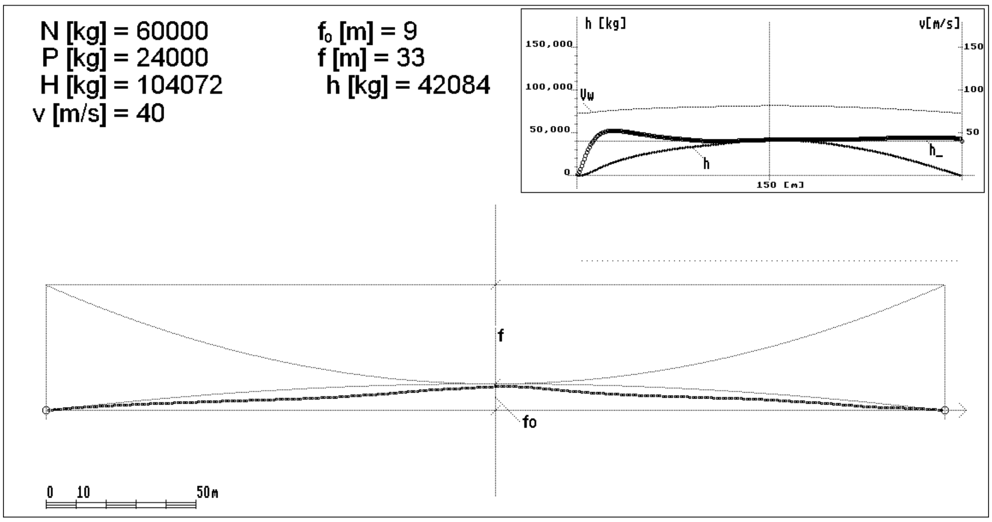
The solutions for different load conditions and different values of the velocity v of the load are given in Appendix B. The corresponding trajectories of the load P are given in Fig.8a, b, c.



8a




8b




8c

Fig. 8a, b, c. represents the trajectory of the load *P* obtained from the numerical solution of eq.(12). We also plot versus x, in the inserted box, the function *h(t)* (representing the increase of the traction stress due to the load), together with its approximate expression h and to the propagation velocity $v_w$ of a perturbation along the bridge. (J = $2\cdot10^{-3}$ [$m^4$], EJ [$Nw\cdot m^2$] = $4.116\cdot10^8$). The function h_ is the first term of t series expansion (see appendix B).

The vehicle trajectories obtained by the present theory neglecting damping terms, are quite different from the ones obtained in quasi-static conditions. They are closer to the loadless girder profile, but submitted to a waving behaviour, which is strongly reduced by taking semi-active and active dampers into account (see fig.4).

**4 - Conclusions**

The present work has validated Müller's approach [2]. In quasi-static conditions, indeed, the curvature of the girder turns out to be almost uninfluent, but the advantages of Müller's idea become evident when the dynamic behaviour is considered. The curvature increases the rigidity of the structure, by increasing the load on the sustaining ropes. This is a positive feature, since the weight of the structure alone would be too much light. Although the dynamical behaviour was not fully exploited in the present work, the basic numerical tools are already available.

We have verified that a structure such as the one considered here may undergo, when loaded, very large deformations, which however provide the necessary working conditions. Just as in the case of cableways, a strong deformation of the rope is required in order to sustain the load. The vertical oscillations of the girder must be reduced by the suitable semi-active dampers of each hanger.

The urban transportation (and suburban connection) system proposed here may be advantageously competitive with respect to the present day transportation systems.

This article was done only a preliminary approach to the complete design of a new transport system, an approach based on a discussion approximate math, ignoring some important aspects of the whole system vibrations. The hollow rail is equivalent to the girder of a normal suspension bridge. The required rigidity of this girder is obtained by the great tensile stress N. The lightness of this particular deck requires very light support cables. And 'so possible to connect large distances with very fast vehicles without being affected by adverse weather conditions.

## REFERENCES


1) Lee CH, Kim CW, Kawatani M, Nishimura N, Kamizono T. Dynamic response analysis of monorail bridges under moving trains and riding confort of trains, Engineering Structures 2005;27:1999-2013.
2) Müller G, Cableway arrangement, US Patent 3,753,406 (1973)
3) Baltensperger R, Pfister HB, Suspended rail structure especially for monorail vehicles, US Patent 4,208,969 (1980)
4) Dalliard F, Suspended motorised vehicle, US Patent 4,641,587 (1987)
5) Lamoreaux B, Wettstein H, Aasheim P, Pugin AO. Elevated cableway system, US Patent 6,606,954 B1 (2003)
6) Project ACE, Institute of Fluid Power, University of Technology, Dresden. Active Control in Civil Engineering, Brite Euram 3, BRPR-CT 97-0402.
7) Belluzzi O Scienza delle Costruzioni, vol. IV, chap. XXXIV, Zanichelli, Bologna 1955
8) Kármán T, Biot MA. Mathematical Methods in Engineering (*Metodi matematici nell'ingegneria* chap. VII, Edizioni scientifiche Einaudi - 1951)
9) Flügge W. Handbook of engineering mechanics, Chap. 30, McGraw-Hill 1962
10) Brownjohn JMW. Vibration characteristics of a suspension footbridge. Journal of Sound and Vibrations 1997; 202, 29-46

11) Pirner M, Aeroelastic characteristics of a stressed ribbon pedestrian bridge spanning 252 m. Journal of Wind Engineering and Industrial Aerodynamics 1994; 53, 301-14

12) Inglis C. E. A mathematical treatise on vibrations in railway bridges, Cambridge, University Press,1934.

**Appendix A**

The solution of eq.(10) may be shown to be given by the function

$$w(x) = w_1(x) - \frac{h}{H}\ (q + q_N)\, w_2(x) \qquad \text{(A1)}$$

where the functions $w_1(x)$ and $w_2(x)$ are, respectively, the solutions of the equations

$$E\,J\,\frac{d^4 w}{dx^4} - (H + h + N)\,\frac{d^2 w}{dx^2} = p(x)$$

$$E\,J\,\frac{d^4 w}{dx^4} - (H + h + N)\,\frac{d^2 w}{dx^2} = 1 \qquad \text{(A2)}$$

The function $w_1(x)$ may be expressed by means of a suitable series expansion [...] in the form:

$$w_1(x) = \frac{2\,P\,L^3}{\pi^4 E\,J} \sum_{n=1}^{\infty} \sin(\frac{n\pi a}{L})\ \frac{\sin(\frac{n\pi x}{L})}{n^2\ (n^2 + \frac{N_t L^3}{\pi^2 E\,J})} \qquad \text{(A3)}$$

where $N_t = H+h+N$ , $L$ is the length of the bridge and $a$ is the distance of the load $P$ from $x = 0$. The function $w_2(x)$, in its turn, may be given in the form:

$$w_2(x) = \frac{2\ L^4}{\pi^5 E\,J} \sum_{n=1}^{\infty} [1 - \cos(n\pi)]\ \frac{\sin(\frac{n\pi x}{L})}{n^3\ (n^2 + \frac{N_t L^3}{\pi^2 E\,J})} \qquad \text{(A4)}$$

**Appendix B**

Putting now: $h(t) = \boldsymbol{h} \sin(\omega\, t)$, and observing that

$$\frac{q + q_N}{H} = 2\,\frac{q + q_N}{H\pi} \sum_{n=1}^{\infty}\ \sin(\frac{n\,\pi\,x}{L})\,\frac{1 - \cos(n\,\pi)}{n}$$

we may write now a particular solution of eq.(12) in the form

$$w_1(x,t)=\sum_{n=1}^{\infty} a_n \sin(\frac{n\pi x}{L}) \sin(n\omega t) \quad . \qquad \text{(B1)}$$

The coefficients $a_n$ may be obtained by inserting the function (B1) into eq.(12), thus obtaining

$$a_n = \frac{p_n - 2\,\boldsymbol{h}\,\dfrac{q+q_n}{H\,\pi\,n}(1-\cos(n\pi))}{EJ\,(\dfrac{n\pi}{L})^4 + N_t(\dfrac{n\pi}{L})^2 - m\,n^2\omega^2} \qquad \text{(B2)}$$

The general solution of eq.(12) may be expressed in the form

$$w(x,t) = w_1(x,t)+ w_2(x,t), \qquad \text{(B3)}$$

by adding the particular solution (B1) to the general solution

$$w_2(x,t)=\sum_{n=1}^{\infty} c_n \sin(\frac{n\pi x}{L}) \sin(\omega_n\, t) \qquad \text{(B4)}$$

of the homogeneous equation:

$$EJ\,\frac{d^4 w}{dx^4} - (H+N+h(t))\,\frac{d^2 w}{dx^2} + m\,\frac{d^2 w}{d\,t^2} = 0$$

where the function h(t) is provided by eq. (7), which is considered to hold also in transient behaviour. In eq.(B4), $\omega_n$, obtained from eq.(9), represents the free oscillation frequencies of the entire structure, and the coefficients $c_n$ may be directly obtained from the initial conditions: $w(x, t=0)= w_o(x)$ and: $\dfrac{\partial w(x,t=0)}{\partial t}=0$, thus obtaining:

$$c_n = \frac{n\,\omega}{\omega_n} a_n \qquad \text{(B5)}$$

The frequency $\omega_n$ must be modified [12] in order to take into account the mass P/g of the vehicle, according to the relation

$$\omega_n^{\,2} = \frac{n^2\pi^2}{L^4}\left[\frac{n^2\pi^2\;E\;J}{m+\dfrac{2P}{g\,L}\sin^2(\omega t)} + \frac{N_t\,L^2}{m}\right]$$

Inserting now into eqs. (B1) and (B4) the expressions of $a_n$ and $c_n$ provided, respectively, by eqs.(B2) and (B5), we finally get the detailed expression of the general solution (B3) of eq. (12).

## Appendix C

**Engineering** The main technical difficulty to realize this type of bridge is encountered in putting in traction the hollow rail (Figs. 10,11), consisting of two locked steel coil ropes (g), connected by a body of welded sheets. The state of permanent tensile stress is obtained by applying a near constant horizontal force at one end of the rail beam. To apply the tensile stress N to this extreme of the hollow rail, this end must necessarily be movable when the load and the temperature varies. For $\Delta t$ = 30 °C a bridge length of 3300 meters (the length of the of the Messina bridge design), has a displacement of 1.22 [m] ,

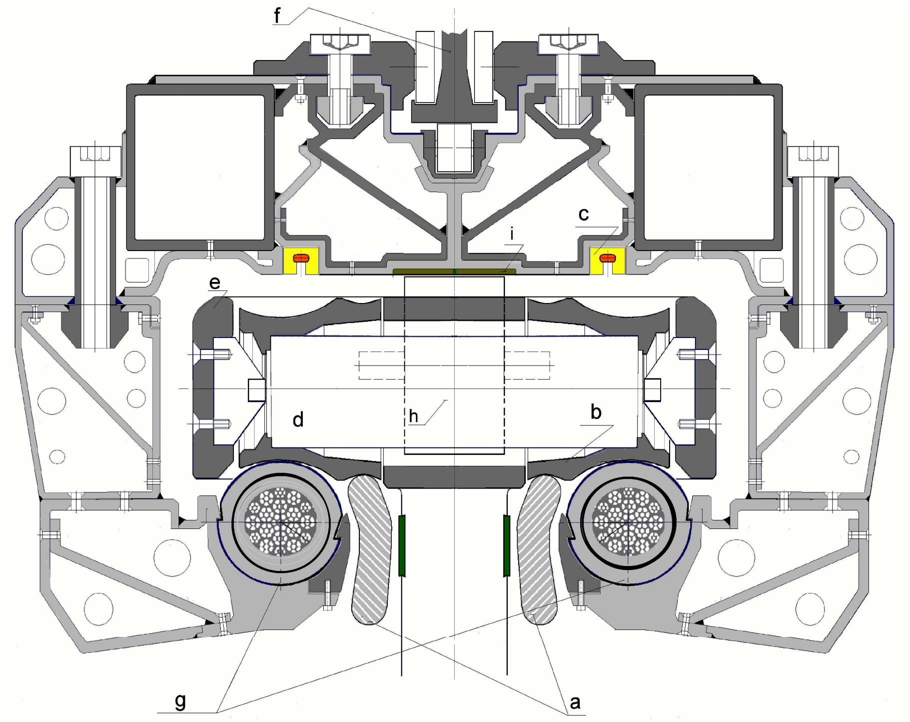



Fig. **9** Details of the hollow rail containig the boogie (e) of the vehicle: **a**) two short additional rigid rails, which are present only where the wire ropes (g) are attached to the traction cylinders (see Fig. 11). The vehicle and its boogies are lifted over the cylindrical rails (in Fig. 11 is shown the passing over of the coupling area of the hydraulic traction cylinders); **b**) grooved wheel to support the weight of the vehicle; **c**) two partly insulated power electric line are inserted in the upper side; **d**) rotating electric motor cage, containing also the gear reduction; **e**) chassis of the boogie; **f**) upper sliding anchor that connects the hollow rail on the above main bridge wire ropes, fixed at the top of the support pylons (see fig.10); **g**) two cylindrical rails consisits of two locked coil ropes; **h**) rubberized traction wheel which acts on the upper rubber tape (i).

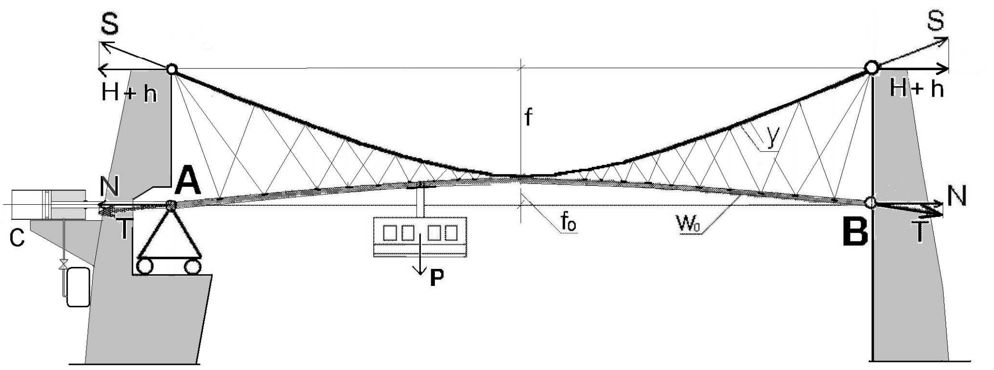



Fig. 10- Scheme of the bridge span with the traction cylinders C.

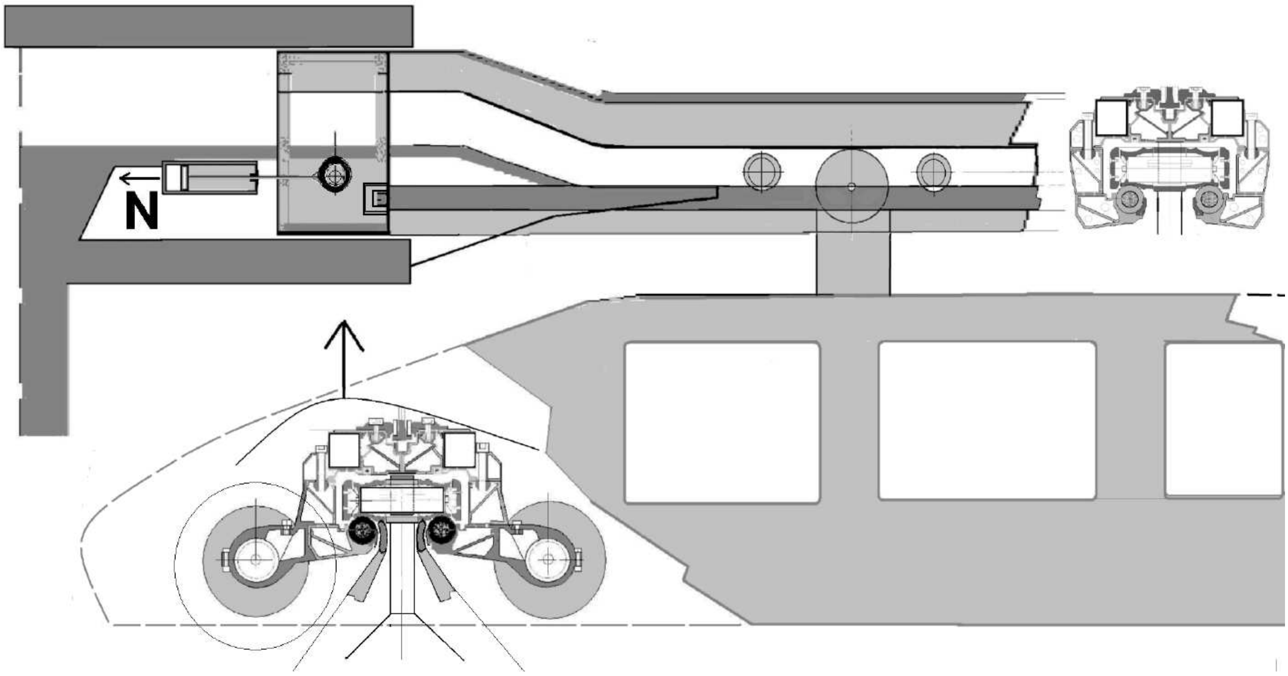



Fig. 11 - Schematic view of the system (side A in Fig. 10) to climb over the attachment point of the hydraulic cylinders for pulling the hollow-beam rail.

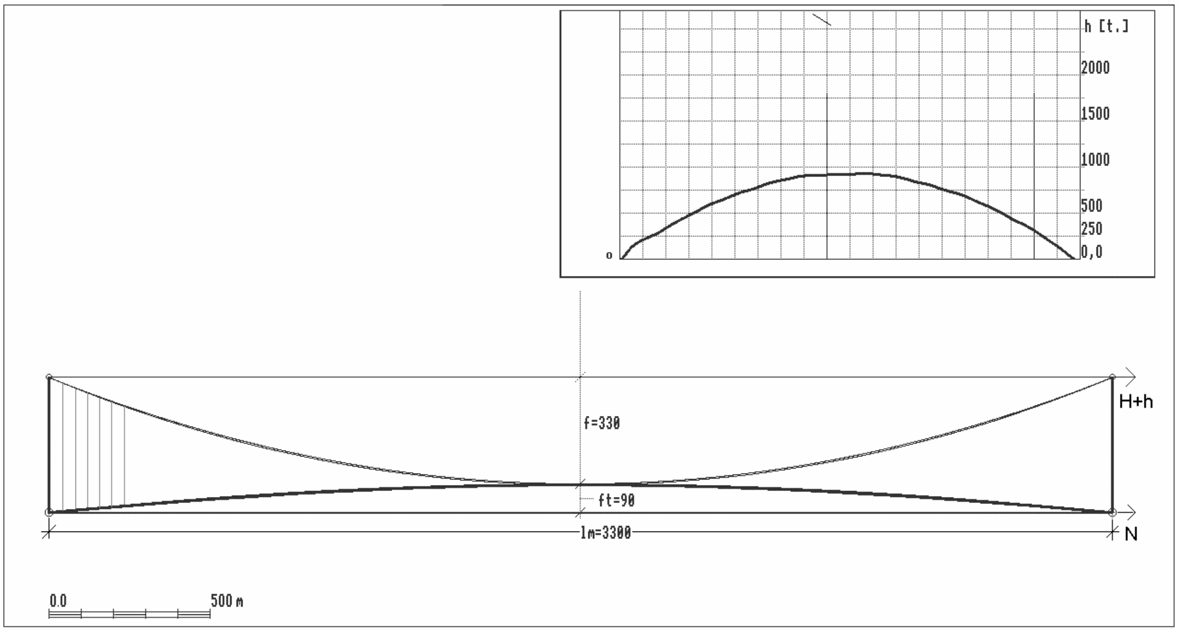



Fig. 12-a. - N=**8.6 $10^3$** [t]; $h_{max}$= 0.9345 $10^3$ [t]; vehicle velocity: v=30 [m/s]; vehicle weight: 440 [t]

Each cylinder exerts a traction equal to N/2 = 4800 [t]. With a fluid pressure equal to: 200 [kg /$cm^2$] each cylinder must have a working area equal to: 24000 $cm^2$ - hydraulic cylinder Internal diameter:180 [cm]; Piston rod diameter: 20 [cm]; working area: 25,000[$cm^2$]

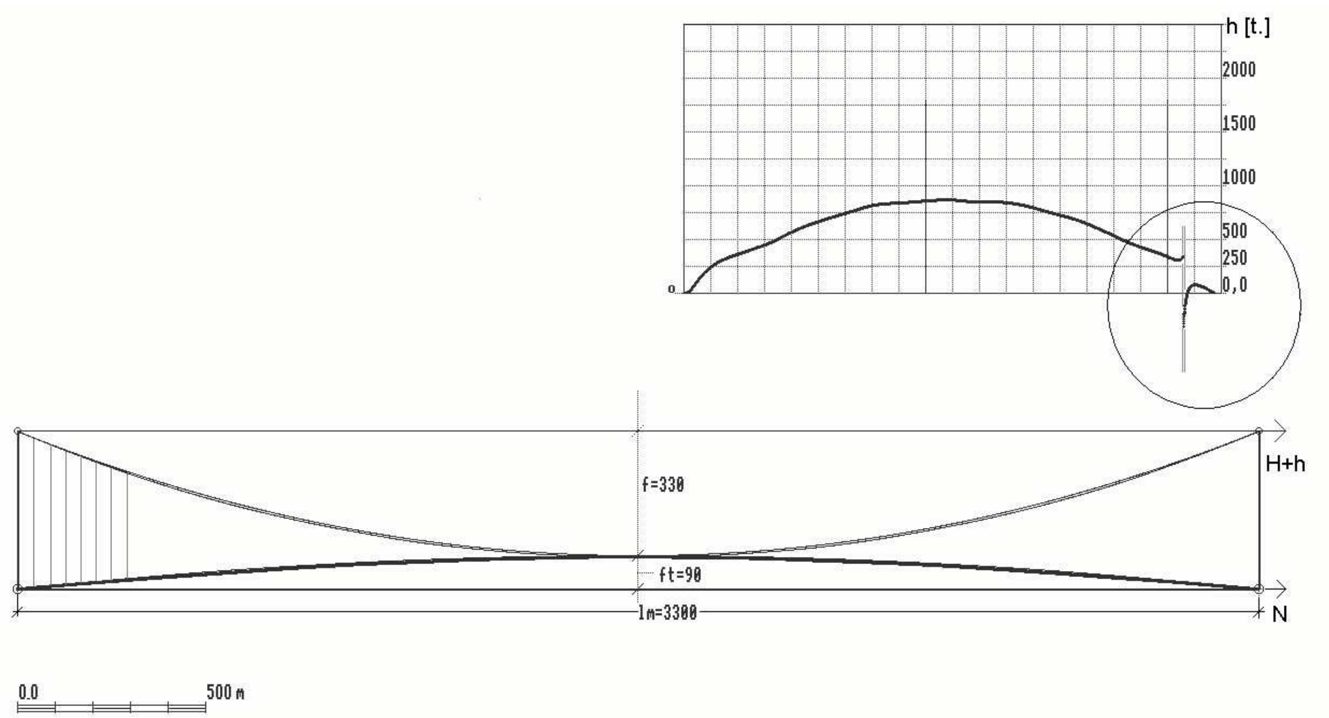



Fig. 12-b.  N=4600 [t] the structure collapses.

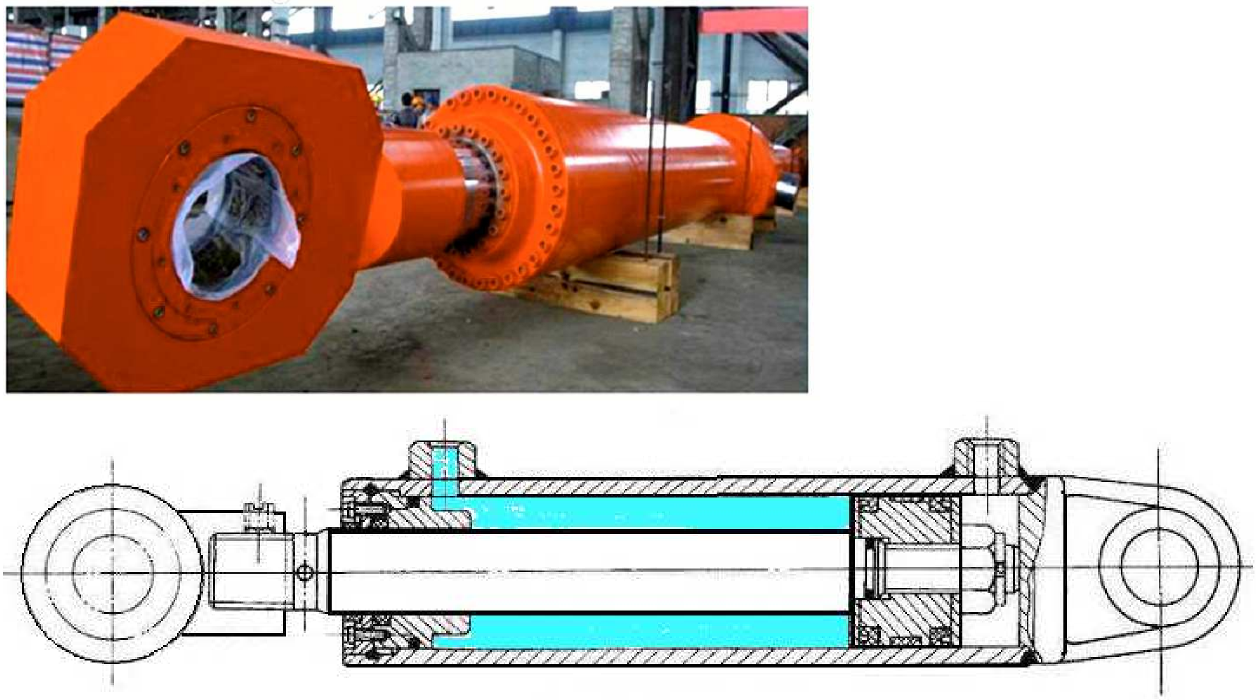


Fig. 13. Hydraulic cylinder with dimensions compatible with those of the project (Messina bridge).

The hollow rail constitutes the deck (girder) of a suspension bridge.  To obtain the necessary rigidity of this girder it is necessary to apply a large tensile stress N.

**FIGURE CAPTIONS**

Fig. 1. Scheme of Müller's system [2]. The critical points (A) of such a system are the ones where the rail (1) is held at a fixed height, thus opposing to the sustaining rope (2).

Fig. 2. Artistic view of a transportation system based on Müller's project, with a pylon containing a vehicle stop.

Fig. 3. General view of the left half of the bridge, submitted to the stresses H,N,$S_{max}$. The positions a) indicate the active dampers [6]. The hangers are in clined at 45°, but twined in such a way that the only component of the suspension force at the points of sliding anchorage is normal to the girder (the theory is carried out considering the vertical hangers).

Fig. 4. Configuration of the structure imposed by its weight ($q(x)= q \cong$ const) and by the load $q_N$ , in the absence of an externally imposed load

Fig. 5. Configuration of the structure after the application of the load (at x = a).
The configuration in the absence of load is represented by the heavy line; the light line represents the shape of the structure deformed by the load P.

Fig. 6a, b, c. The configuration of the whole structure is obtained (in static conditions) from eqs. 6), A3), A4) and 7) for some positions of the external load P and for different values of the horizontal component, N, of the tension T applied at the ends of the girder rail. $E\cdot J = 1.85\cdot10^8$ [$Nw\cdot m^2$]

Fig. 7a, b. The quasi-static trajectory of the load is obtained by repeating the calculation of Fig.1 for a set of load positions all along the girder ($E\cdot J = 1.85\cdot10^8$ [$Nw\ m^2$]).
Fig. 8 a, b, c. represents the trajectory of the load *P* obtained from the numerical solution of eq.(12). We also plot versus x, in the inserted box, the function *h(t)* (representing the increase of the traction stress due to the load), together with its approximate expression h and to the propagation velocity $v_w$ of a perturbation along the bridge. ($J = 2\cdot10^{-3}$ [$m^4$], $EJ$ [$Nw\cdot m^2$] $= 4.116\cdot10^8$)

Fig. 9 Details of the hollow rail containig the boogie (e) of the vehicle: a) two short additional rigid rails, which are present only where the wire ropes (g) are attached to the traction cylinders (see Fig. 11). The vehicle and its boogies are lifted over the cylindrical rails (in Fig. 11 is shown the passing over of the coupling area of the hydraulic traction cylinders); b) grooved wheel to support the weight of the vehicle; c) two partly insulated power electric line are inserted in the upper side; d) rotating electric motor cage, containing also the gear reduction; e) chassis of the boogie; f) upper sliding anchor that connects the hollow rail on the above main bridge wire ropes, fixed at the top of the support pylons (see fig.10); g) two cylindrical rails consists of two locked coil ropes; h) rubberized traction wheel which acts on the upper rubber tape (i).

Fig. 10- Scheme of the bridge span

Fig. 11 - Schematic view of the system (side A in Fig. 10) to climb over the attachment point of the hydraulic cylinders for pulling the hollow-beam rail.

Fig. 12-a. N=8.6 $10^3$ [t]; $h_{max}$= 0.9345 $10^3$ [t]; vehicle velocity: v=30 [m/s]; vehicle weight: 440 [t]

Fig. 12-b. For N=4600 [t] the structure collapses!

Fig.13. Hydraulic cylinder with dimensions compatible with those of the project (Messina bridge).